Equation Tutorial
2009

We present five problems that involve calculations. Annotated answers are given at the end. When performing calculations, it is helpful to always include the units (i.e., mls, liters, mls/min, etc) of the values you enter into the equation and, after appropriate cancellation, insure that your answer is in the desired units.

Problem #1. A subject is asked to take a 500 ml breath of air from FRC then hold it at that volume. During the maneuver, intrapleural pressure measured with an esophageal catheter changes from -5 cm at FRC to -15 cm at the end of the inspiration. Calculate the lung compliance and indicate which of the following diseases is most compatible with your result (normal \( C_L \) is 0.2 L/cmH2O).

A. emphysema
B. fibrosis
C. chronic bronchitis
D. muscle weakness

ANSWER

Use the equation

\[
C_L = \frac{\Delta V}{\Delta P} = \frac{\Delta V}{(P_A - P_{PL}) - (P_A - P_{PL})}
\]

end inspiration  end expiration

\( \Delta V \) is the change in lung volume and \( \Delta P \) is the change in alveolar distending pressure. The alveolar distending pressure is \( P_A - P_{PL} \) (inside minus outside). Under static conditions (i.e., absence of airflow), \( P_A = 0 \).

\[
C_L = \frac{0.5L}{[0 - (-15 \text{cmH}_2\text{O})] - [0 - (-5 \text{cmH}_2\text{O})]} = \frac{0.5L}{10.0 \text{cmH}_2\text{O}} = 0.05 \text{ L/cmH}_2\text{O}
\]

The \( C_L \) of the patient (0.05L/cmH2O) is lower than normal (0.2L/cmH2O), which is consistent with pulmonary fibrosis (answer B). \( C_L \) is increased in emphysema, and unchanged in chronic bronchitis and muscle weakness.

Problem #2. The ABG of a 40 year old patient indicates a \( P_{aO_2} = 50 \text{ mmHg} \), \( P_{aCO_2} = 55 \text{ mmHg} \), pH = 7.35, [HCO3\text{-}] = 29 mEq/L. Is the patient’s hypoxemia due to alveolar hypoventilation (inadequate movement of air into and out of the lung) and/or V/Q mismatch?

ANSWER

This subject definitely has hypoventilation—we know this because \( P_{aCO_2} \) is above the normal range of 35-45 mmHg. V/Q mismatch (or more precisely regions of low V/Q) as a cause of hypoxemia is determined by looking at the A-a gradient \( (P_{aO_2} - P_{aO_2}) \) for O2. We are given \( P_{aO_2} \), but we need to use the alveolar gas equation to calculate \( P_{aO_2} \):
\[ P_{A02} = P_{102} - \frac{P_{aCO2}}{0.8} \]

Where \( P_{102} = (P_B - P_{H2O}) \times F_{102} \)

\[ P_{102} = (760 - 47\text{mmHg}) \times 0.21 = 150\text{mmHg} \]

\[ P_{A02} = 150 - \frac{55\text{mmHg}}{0.8} = 81\text{mmHg} \]

\[ P_{A02} - P_{a02} = 81 - 50 = 31\text{mmHg} \]

In a perfect lung with no V/Q mismatch or shunt, A-a = 0. All of us have some V/Q mismatch (effect of gravity on V and Q) and shunt (venous admixture from the bronchial circulation) that increases with age. One rule of thumb for A-a is age/3, but not greater than 15 mmHg. The expected A-a in our 40 y.o. patient is ~13 mmHg. Since the calculated A-a of 31 mmHg is larger than expected, we conclude that the patient has a pathological V/Q mismatch. Hence, hypoxemia in this patient is due to both V/Q mismatch and hypoventilation.

On an exam, we could ask you if this patient is more likely to have a neuromuscular problem or COPD. In this case, the correct choice would be COPD, as neuromuscular disease generally presents with hypoventilation, but without V/Q mismatch.

**Problem #3.** A 55 yo female with a history of smoking is admitted to the hospital with respiratory distress. Physical exam reveals bilateral end-expiratory wheezing and hyperresonance on percussion. ABG shows a \( P_{aCO2} \) of 38 mmHg and \( P_{aO2} \) of 88 mmHg. She has a high minute ventilation (\( V_E = 10.8 \text{L/min} \)) with increased rate (18/min) and depth of breathing (\( V_T = 600 \text{mls} \)). The patient’s mixed expired gas was collected over several minutes and was found to have a \( P_{CO2} \) of 16 mmHg.

a. Evaluate the patient’s physiological dead space. Based on your calculation of dead space, indicate whether this patient is more likely to have emphysema or chronic bronchitis.

b. Calculate the patient’s alveolar ventilation

**ANSWER**

a. Use the Bohr equation to calculate \( V_D/V_T \).

\[ \frac{V_{DCO2}}{V_T} = \frac{P_{aCO2} - P_{ECO2}}{P_{aCO2}} \]
\[ \frac{38 \text{ mmHg} - 16 \text{ mmHg}}{38 \text{ mmHg}} = 0.58 \]

Her \( V_D/V_T \) is very high (normal is ~1/3, normal range is 0.2 to 0.4) indicating regions of high V/Q (i.e., wasted ventilation). High V/Q is prominent in emphysema due to destruction of capillaries and formation of bullae from destruction of alveolar walls. The pure bronchitic patient may also have regions of high V/Q, but to a relatively small extent. The physiological consequence of increased dead space is increased work of breathing. If the increased demand for \( \dot{V}_E \) cannot be accommodated, then hypoventilation (increased \( P_{aCO_2} \)) will occur and may rapidly progress to respiratory failure.

B. To calculate alveolar ventilation you need to know the physiological dead space (anatomic + alveolar deadspace), which is 0.58 \( V_T \) from your calculations using the Bohr equation.

\[ V_A = V_T - V_D = V_T - 0.58 V_T = 0.60L - 0.58(0.60L) = 0.25 \text{ L} \]

\[ V_A = V_A \times f = (0.25L)(18/\text{min}) = 4.5 \text{ L/min} \]

Alternatively, since 58% of \( \dot{V}_E \) (total minute ventilation) is dead space ventilation, then 42% of \( \dot{V}_E \) must be alveolar ventilation:

\[ \dot{V}_E = V_T \times f = 10.8 \text{ L/min} \]

\[ V_A = 0.42 \dot{V}_E = 0.42 \times 10.8 \text{ L/min} = 4.5 \text{ L/min}. \]

**Problem #4.** A healthy person has a resting \( V_A \) of 4.3 L/min and a \( \dot{V}_{CO_2} \) of 200 mls/min. During exercise her \( \dot{V}_{CO_2} \) increases to 600 mls/min and her \( P_{aCO_2} \) remains stable at 40 mmHg. What is her \( V_A \) during exercise?

A. 2.15 L/min  
B. 4.30 L/min  
C. 6.8 L/min  
D. 10.1 L/min  
E. 12.9 L/min

**ANSWER**

Answer is E. \( P_{aCO_2} \propto \dot{V}_{CO_2}/V_A \), therefore, if \( \dot{V}_{CO_2} \) increases 3 fold (600 ml/min)/(200 mls/min), and \( P_{aCO_2} \) remains constant, then \( V_A \) must increase 3 fold.

**Problem #5.** A patient with pneumonia has severe hypoxia and is admitted to the hospital. His ABG before administration of supplemental oxygen revealed:

\( P_{aO_2} = 55 \text{ mmHg} \) (\( O_2 \) sat = 0.89)  
\( P_{aCO_2} = 30 \text{ mmHg} \)
\[ [\text{Hb}] = 15 \text{ g Hb/100 ml blood} \]

\[ P_{A02} = 112 \text{ mm Hg} \quad (O_2 \text{ sat} = 0.98) \quad (\text{the presumed} \ P_{A02} \ \text{of ideal alveoli—if we didn’t give you this number you would calculate it using the alveolar gas equation}) \]

A Swan-Ganz catheter inserted into the pulmonary artery additionally provided a 
\[ P_{V02} = 34 \text{ mmHg} \quad (O_2 \text{ sat} = 0.63). \]

Evaluate his physiological shunt.

**ANSWER**

You will use is the shunt equation:

\[ \frac{Q_S}{Q_T} = \frac{C_{cO2} - C_{aO2}}{C_{cO2} - C_{V02}} \]

a. Determining \( O_2 \) content of arterial blood (\( C_{aO2} \)), mixed venous blood (\( C_{V02} \)), and capillary from ideal alveoli (\( C'cO2 \)) requires knowledge of \( %O_2 \) saturation of Hb at each relevant \( P_{O2} \) (we estimated these from the oxyHb dissociation curve).

\( O_2 \) content = (g Hb/100ml blood) \( \times \) (\( O_2 \) sat) \( \times \) (1.34 ml \( O_2 \)/g Hb) + dissolved \( O_2 \)

The dissolved \( O_2 \) is negligible when breathing room air

\( C'cO2 = (15 \text{ g Hb/100ml blood})(0.98)(1.34 \text{ml O}_2/\text{g Hb}) = 19.7 \text{ml O}_2/100\text{ml blood} \)

\( C_{aO2} = (15 \text{ g Hb/100ml blood})(0.89)(1.34 \text{ml O}_2/\text{g Hb}) = 17.89 \text{ml O}_2/100\text{ml blood} \)

\( C_{V02} = (15 \text{ g Hb/100ml blood})(0.63)(1.34 \text{ml O}_2/\text{g Hb}) = 12.66 \text{ml O}_2/100\text{ml blood} \)

b. Enter these numbers into the shunt equation:

\[ \frac{Q_S}{Q_T} = \frac{19.7 - 17.89}{19.7 - 12.66} = 0.26 \]

\( Q_S/Q_T \) is 0.26. I.e., 26\% of his cardiac output behaved as though it received no fresh \( O_2 \) from the inspired air. This occurred because blood went to regions of the lung that were nonventilated or underventilated relative to the rate of perfusion. In pneumonia, these are regions of the lung that receive inadequate ventilation because they are filled with fluid or inflammatory debris or may be atelectic. In a healthy person \( Q_S/Q_T \) is less than 5\%. 

